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Document No. 108 May 15, 1923

Definition of the requirements in

Plane Geometry
Solid Geometry
Plane and Solid Geometry, Major Requirement
Plane and Solid Geometry, Minor Requirement

adopted by the College Entrance Examination Board April 21, 1923 upon the recommendation of the following Commission on College Entrance Requirements in Mathematics:

Professor William F. Osgood, Harvard University, *Chairman*
Mr. Thomas L. Bramhall, High and Latin School, Cambridge, Mass.
Mr. Ralph Beatley, Horace Mann School, New York, N. Y.
Mr. John A. Foberg, Department of Public Instruction, Harrisburg, Pa.
Professor Dunham Jackson, University of Minnesota
Professor William R. Longley, Yale University
Mr. Islay F. McCormick, Albany Academy, Albany, N. Y.
Professor Helen A. Merrill, Wellesley College
Professor David Eugene Smith, Teachers College, Columbia University
Professor Harry W. Tyler, Massachusetts Institute of Technology
Dr. Eula A. Weeks, Cleveland High School, St. Louis, Mo.

Upon receipt of ten cents, which may be remitted in postage, a copy of this document will be sent to any address. Upon application, a copy will be sent without charge to any teacher of Mathematics.

The Commission's Report on Elementary Algebra, Advanced Algebra, and Plane Trigonometry is given in a separate document.

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SYLLABI IN GEOMETRY

The following syllabi have been prepared to indicate the scope of the requirements in geometry. It will be seen that the content of the standard requirements in plane geometry and in solid geometry is essentially the same as heretofore. A redistribution of emphasis on the various parts of the work, designed to lighten the demands on the candidate's memory and to give increased opportunity for attention to the development of geometrical understanding, is explained in subsequent paragraphs of this introductory statement.

To meet the situation created by the gradual disappearance of solid geometry from the schools, the present requirements (Mathematics *C, D,* and *CD*) have been so modified as to permit teachers greater freedom in the treatment of the material, and a new requirement (Mathematics *cd*), covering selected parts of plane and solid geometry and calling for about as much preparation as is represented by the present requirement in plane geometry, has been added.

As in the past, the examinations will consist partly of questions on *book propositions* and partly of *originals.* In the former type of question, the candidate will be asked to give demonstrations of standard theorems which are assumed to have been presented to him in his course of study, or to reproduce standard constructions. Under the latter type are included the demonstration of theorems which are not assumed to be familiar to the candidate, problems of measurement and calculation, and problems in the working out of unfamiliar constructions and the identification of unfamiliar loci. Questions calling for simple geometrical knowledge and understanding may fall under either type.

An important novelty in the present statement of requirements relates to the treatment of book propositions. These will be chosen, not from the entire syllabus at large, but from a restricted list of propositions, designated in each syllabus by asterisks. The starred propositions have been chosen partly with regard to their intrinsic importance, but partly also with regard to their availability for examination purposes. A theorem may be of the highest importance, and yet may have a proof which, for one reason or another, is of such a nature that little information as to the candidate's ability is obtained by asking him to reproduce it. On this account, some of the most important propositions in the course, such as those on the circumference and area of the circle, are omitted from the starred list.

In the case of the unstarred propositions, the candidate will be expected to be familiar with their content, so as to be able to answer questions about their substance or use them as a basis for solving originals, but he will not be assumed to have their demonstrations in mind. A few propositions have been included on account of their general importance and interest without reference to direct or indirect use for examination purposes. The demonstration of an unstarred proposition may sometimes be called for on an examination, but only if it is of such a nature that the candidate can reasonably be expected to work it out as an original. It will undoubtedly be desirable to present the proofs of most of the unstarred propositions in regular sequence in the course, formally or informally, as one of the best supplements to the instruction directly needed for the satisfaction of the minimum requirements. It is intended, however, that it shall be most advantageous for the pupil, in preparing for the examinations, to spend his time in developing facility in independent thinking on geometrical topics, rather than in memorizing such proofs.

About one-third of the examination will be devoted to book propositions from the starred list, except in the case of solid geometry as explained later. The remainder will be divided between easy originals which ought to be within the reach of any properly trained candidate, and originals which, though still not of excessive difficulty, are designed to test the powers of the better candidates. The easy as well as the more difficult originals may be unfamiliar theorems, or problems in mensuration, construction, or loci. Whether papers so planned are found to be intrinsically easier or harder than those that have been set in the past, it is intended that the marking shall be so administered that the difficulty of obtaining a passing grade is not materially changed.

The questions constituting the examination (that is, the main subdivisions of the paper, numbered from 1 to 6, more or less) will as a rule be weighted equally by the readers; but a bonus may be given for exceptional work on a given question, and, on the other hand, the general character of a candidate's paper may indicate serious deficiencies, even though a considerable total be scraped together from his fragmentary answers to many questions. The readers will appraise a book as a whole, and not cling to any mechanical method of marking when occasion demands the exercise of human judgment. The questions may be answered in any order, and it is advisable that the candidate make sure of those which he can answer best. When a single question is composed of two or more parts, the relative weights to be assigned to the parts will be determined by the readers according to circumstances.

Each of the following syllabi has been arranged in what is regarded as a practicable teaching order, but this order is in no wise prescribed. The candidate may base his answers on any sequence of propositions which is logically sound, and may even quote propositions not listed in the syllabus at all, if they have been presented as standard propositions in his course. The readers occasionally have to use their discretion in deciding whether an answer made unduly easy by the citation of an unusual reason deserves full credit. For example, if a candidate proves the starred theorem *23 in Plane Geometry merely by deducing it from theorem 24 or 25, he may reasonably be expected to make up for a considerable saving of time here by a higher standard of performance on the rest of the paper than would otherwise be required.

For convenience of reference, formulas have been appended to some of the theorems in mensuration. The notation used in these formulas is not prescribed, but if the candidate uses a different notation, he should be sure that its meaning is unmistakable.

It is to be emphasized once more that the syllabi have been prepared as a basis for the conduct of examinations, not as a recommendation for the content and arrangement of ideal courses. Many of their features, especially in connection with the distribution of asterisks, have been determined by the peculiar exigencies of examination technique.

In particular, it will be noticed that proofs involving the theory of limits are not required for examination purposes. This is for the reason that the definition and maintenance of a satisfactory standard of performance is difficult for examiners and readers, as well as for teachers and candidates. Apart from the examination requirement, it is in the interest of sound mathematical training that each class of pupils receive the best and most thorough instruction in the theory of limits that they can accept. An immeasurably improved understanding of the meaning of limits has been perhaps the most important achievement of mathematical science in the past hundred years. As this improved knowledge becomes generally available, the importance of the subject for elementary instruction will increase rather than diminish. In the light of developments already in progress, there is reason for the belief that the present period of transition, in which the setting of any kind of uniform standards appears impracticable, will not be of long duration.

PLANE GEOMETRY

The significance of the starred and the unstarred propositions has been explained above.

The originals on the examination will in general depend for their solution on propositions mentioned in the syllabus. But occasionally an original will be so framed that a solution will occur more readily to the candidate who is familiar with such important geometrical facts as the properties of the 30° and the 45° right triangles.

With regard to constructions, the candidate is expected to be able to perform and to describe accurately those listed at the end of the syllabus, and also, as originals, others based on these. He is not required to give proofs of constructions, unless a proof is specifically called for by the question, and such proofs will not be regarded as constituting a part of the book-work requirement, but will have the status of originals. The candidate is expected to be provided with ruler and compasses, and to use them. In default of these instruments, however, he will receive credit for a satisfactory free-hand sketch showing all construction lines.

SOLID GEOMETRY

The work in solid geometry may well be supplemented by a considerable amount of material which remains outside the scope of the examination requirement. The importance of limit proofs has already been emphasized. Other valuable material is contained in the theorems of Cavalieri and Euler, and the so-called prismoid or prismatoid formula.

A few theorems on similar solids have been included in the syllabus, unstarred. The candidate should be familiar with their substance, as a basis for problems in measurement and calculation, but they should not be made the occasion of a large amount of demonstrational work.

The candidate should know what is meant by the perpendicularity of skew lines, but will not be required to deal with the measure of an angle between two skew lines which are not perpendicular to each other.

Attention is again called to the fact that the order in which the propositions are listed is in no wise to be regarded as a prescription of the order in which the proofs shall be taken up. Thus, for example, the sphere may be treated early in the course, and the treatment of dihedral angles based on the geometry thus developed.

The Board wishes to accord all due latitude in the treatment of the subject of solid geometry. It recognizes the value of the further training in logical demonstration which supplements the study of plane geometry and is given in standard courses at the present time. It recognizes also that the intuitive geometry of the early school course may well be carried further as regards both a firmer grasp on space relations and the visualization of space figures, and the mensuration of surfaces and solids in space.

The examinations will be constructed with reference to this larger interpretation of the requirement. In the past, the candidate has been expected to answer six questions, and this will be assumed for convenience in defining the nature of the new examinations. These papers will consist of seven questions, of which the candidate will be expected to answer six. Two of these questions will call for demonstrations of propositions from the starred list, but not both of these propositions will be chosen from Book VI. Many teachers have felt that the amount of formal demonstration demanded by this Book has been excessive and has obscured the subject matter. The purpose of the new requirement is to give the teacher freer hand, enabling him, if he so desires, to teach the facts concerning the relations of lines and planes in space by means of problems and constructions.

In connection with the drawing and measurement of geometrical solids, the candidate should be thoroughly familiar with the terminology of the subject, including such terms as the following; polyhedron; regular polyhedron; diagonal; face diagonal; prism; right prism; oblique prism; truncated prism; parallelepiped; right parallelepiped; rectangular parallelepiped; pyramid; frustum of a pyramid; vertex; lateral edge; slant height; altitude; cylinder and cone, and the customary terms relating to them, defined with complete generality, though restricted in problems of mensuration to the right circular cylinder and cone; zone, lune, great circle, small circle, segment of one base and sector of a sphere.

Problems of mensuration, stated in words or in terms of a figure, form an important topic of the course. The candidate is expected to be familiar with the fundamental formulas of mensuration in plane geometry, with the use of natural sines, cosines, and tangents in solving right triangles, as taught in numerical trigonometry, and with the formulas of solid geometry enumerated in the Appendix to the Syllabus.

Another topic explicitly included is the solution of locus problems, to the extent of visualizing, describing, and representing the figure on paper; formal proofs are not in general required.

With the considerable reduction in the formal proofs demanded by the present requirement, it is expected that candidates will show greater facility in visualizing space figures and in representing such figures on paper. Moreover, a firmer grasp of the facts of solid geometry is contemplated, and problems in mensuration which test all of these things will form a part of the examinations. The type of training expected of a candidate, so far as this new material is concerned, is indicated by a list of problems in the Appendix to the Syllabus.

PLANE AND SOLID GEOMETRY

MAJOR REQUIREMENT

This requirement is based on the syllabi for the two preceding requirements, taken together, and corresponds to the present requirement, Mathematics *CD,* 1½ units.

PLANE AND SOLID GEOMETRY

MINOR REQUIREMENT

This requirement is designed to cover the most important parts of plane and solid geometry, in such a way that the preparation for it can be completed in the time usually devoted to the standard requirement in plane geometry. It is designated as Mathematics *cd,* and counts 1 unit.

The notes bearing on the syllabi for the preceding requirements, with regard to originals and constructions and the matter of skew lines, are applicable here.

A number of theorems have been omitted from the present syllabus which would be used in proving some of the unstarred theorems that are included, so that the latter theorems will necessarily be treated somewhat informally, if the course is not to be made unduly long. The preliminary propositions that are needed for proving the starred theorems have been retained.

An omitted theorem may be used as an original on an examination, if it is of suitable difficulty. And an original may involve a simple omitted theorem incidentally—for example, the corollary that, if two parallel lines are cut by a transversal, the interior angles on the same side of the transversal are supplementary—but then the fact that the candidate has to recognize and prove the point in question will be taken into account in estimating the difficulty of the problem.

PLANE GEOMETRY

BOOK I

TRIANGLES, PERPENDICULARS, AND PARALLELS

Triangles

1*. Two triangles are congruent if two sides and the included angle of one are equal respectively to two sides and the included angle of the other.

2*. Two triangles are congruent if a side and the adjacent angles of one are equal respectively to a side and the adjacent angles of the other.

3*. If two sides of a triangle are equal, the angles opposite these sides are equal.

4*. If two angles of a triangle are equal, the sides opposite these angles are equal.

5*. Two triangles are congruent if the three sides of one are equal respectively to the three sides of the other.

Parallels and Perpendiculars

6*. Not more than one perpendicular can be drawn to a given line from a given external point.

7. Two lines in the same plane perpendicular to the same line are parallel.

8. If a line is perpendicular to one of two parallel lines, it is perpendicular to the other also.

9. Two right triangles are congruent if the hypotenuse and an adjacent angle of one are equal respectively to the hypotenuse and an adjacent angle of the other.

10*. Two right triangles are congruent if the hypotenuse and a side of one are equal respectively to the hypotenuse and a side of the other.

11*. If two parallel lines are cut by a transversal, the alternate interior angles are equal.

12. If two parallel lines are cut by a transversal, the corresponding angles are equal.

13. If two parallel lines are cut by a transversal, the interior angles on the same side of the transversal are supplementary.

14. If two lines are cut by a transversal so that a pair of alternate interior angles are equal, the lines are parallel.

15. If two lines are cut by a transversal so that a pair of corresponding angles are equal, the lines are parallel.

16. If two lines are cut by a transversal so that two interior angles on the same side of the transversal are supplementary, the lines are parallel.

17. If two angles have their sides respectively parallel, or respectively perpendicular, they are either equal or supplementary.

Parallelograms

18. The opposite sides of a parallelogram are equal and the opposite angles are equal.

19. The diagonals of a parallelogram bisect each other.

20*. If the opposite sides of a quadrilateral are equal, the figure is a parallelogram.

21*. If two sides of a quadrilateral are equal and parallel, the figure is a parallelogram.

22*. If three or more parallels intercept equal segments on one transversal, they intercept equal segments on every transversal.

Sum of the Angles of a Triangle

23*. The sum of the three angles of a triangle is equal to two right angles.

24. An exterior angle of a triangle is equal to the sum of the two opposite interior angles.

25. The sum of the angles of a convex polygon of n sides is $2(n-2)$ right angles.

Inequalities

26. If two sides of a triangle are unequal, the angles opposite these sides are unequal, and the angle opposite the greater side is the greater.

27. If two angles of a triangle are unequal, the sides opposite these angles are unequal, and the side opposite the greater angle is the greater.

28. If two sides of one triangle are equal respectively to two sides of another triangle, but the included angle of the first is greater than the included angle of the second, then the third side of the first is greater than the third side of the second.

29. If two sides of one triangle are equal respectively to two sides of another triangle, but the third side of the first is greater than the third side of the second, then the angle opposite the third side of the first is greater than the angle opposite the third side of the second.

Loci

30*. The locus of points equidistant from two given points is the perpendicular bisector of the line joining them.

31*. The locus of points equidistant from two given intersecting lines is the pair of lines bisecting the angles formed by the given lines.

Lines through a Point

32. The perpendicular bisectors of the sides of a triangle meet in a point.

33. The bisectors of the angles of a triangle meet in a point.

34. The altitudes of a triangle meet in a point.

35. The medians of a triangle intersect in a point, which is two-thirds of the distance from any vertex to the mid-point of the opposite side.

BOOK II

THE CIRCLE

36*. Through any three given points not lying in a straight line one circle, and only one, can be drawn.

37. In the same circle, or in equal circles, equal central angles intercept equal arcs, and equal arcs determine equal central angles.

38. In the same circle, or in equal circles, two central angles are proportional to their intercepted arcs.

39. In the same circle, or in equal circles, if two arcs are equal, their chords are equal; and, conversely, equal chords determine equal minor arcs and equal major arcs.

40*. In the same circle, or in equal circles, equal chords are equidistant from the center, and chords equidistant from the center are equal.

41*. A diameter perpendicular to a chord of a circle bisects the chord and the arcs determined by the chord.

42. A diameter which bisects a chord (that is not a diameter) is perpendicular to it.

Tangents

43. A line perpendicular to a radius at its outer extremity is tangent to the circle.

44. The tangent to a circle at a given point is perpendicular to the radius drawn to that point.

45. If tangents to a circle from an external point are drawn, they make equal angles with the line joining the given point to the center, and their segments from the given point to the points of contact are equal.

46. Two parallel secants, or two parallel tangents, or a tangent and a secant parallel to it, intercept equal arcs on a circle.

The Measure of Angles

47*. An inscribed angle is measured by half the intercepted arc.

48. Angles inscribed in the same segment, or in equal segments, are equal.

49. An angle inscribed in a semicircle is a right angle.

50. An angle formed by a tangent and a chord of a circle is measured by half the intercepted arc.

51. An angle formed by two intersecting chords of a circle is measured by half the sum of the intercepted arcs.

52. An angle formed by two secants of a circle, or by two tangents, or by a secant and a tangent, intersecting at a point outside

the circle, is measured by half the difference between the intercepted arcs.

Inequalities

53. If in the same circle, or in equal circles, two minor arcs are unequal, the greater arc determines the greater chord.

54. If in the same circle, or in equal circles, two chords are unequal, the greater chord determines the greater minor arc.

55. If in the same circle, or in equal circles, two chords are unequal, the shorter is at the greater distance from the center.

56. If in the same circle, or in equal circles, two chords are unequally distant from the center, the one at the greater distance from the center is the shorter.

BOOK III

SIMILAR POLYGONS

Similar Triangles

57. A line parallel to one side of a triangle and intersecting the other two sides divides these two sides proportionally.

58*. If a line divides two sides of a triangle proportionally, it is parallel to the third side.

59*. Two triangles are similar if the angles of one are respectively equal to the angles of the other; or if two angles of one are respectively equal to two angles of the other.

60*. Two triangles are similar, if an angle of one equals an angle of the other and the sides including these angles are proportional.

61*. Two triangles are similar if their sides are respectively proportional.

Applications

62. The bisector of an interior angle of a triangle divides the opposite side into segments which are proportional to the adjacent sides.

63. The bisector of an exterior angle of a triangle divides the opposite side externally into segments proportional to the adjacent sides.

64. The segments cut off on two transversals by a series of parallels are proportional.

65. In any right triangle, the perpendicular dropped from the vertex of the right angle to the hypotenuse divides the triangle into two triangles, each similar to the given triangle.

66*. In any right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

67*. If two chords intersect in a circle, the product of the segments of one is equal to the product of the segments of the other.

68. If from a point outside a circle a secant and a tangent are drawn, the tangent is the mean proportional between the whole secant and its external segment.

Similar Polygons

69. The perimeters of two similar polygons have the same ratio as any two corresponding sides.

70*. If two polygons are similar, they can be divided into triangles which are similar and similarly placed.

71. If two polygons can be divided into triangles which are similar and similarly placed, the polygons are similar.

BOOK IV

AREAS OF POLYGONS

72*. The area of a parallelogram is equal to the product of its base by its altitude:

$$A = bh.$$

73. The area of a triangle is equal to half the product of its base by its altitude:

$$A = \tfrac{1}{2}bh.$$

74. Two parallelograms, or two triangles, having equal bases and equal altitudes are equivalent.

75*. The area of a trapezoid is equal to half the product of the sum of its bases by its altitude:

$$A = \tfrac{1}{2}(b+b')h.$$

76*. The areas of two similar triangles are to each other as the squares of any two corresponding sides.

77*. The areas of two similar polygons are to each other as the squares of any two corresponding sides.

BOOK V

THE MEASURE OF THE CIRCLE

78*. A circle can be circumscribed about any regular polygon.

79. A circle can be inscribed in any regular polygon.

80. If a circle is divided into equal arcs, the chords of these arcs form a regular inscribed polygon, and the tangents at the points of division form a regular circumscribed polygon.

81*. The area of a regular polygon is equal to half the product of its apothem by its perimeter.

82. Regular polygons of the same number of sides are similar.

83. The perimeters of two regular polygons of the same number of sides are to each other as the radii of the inscribed circles, or as the radii of the circumscribed circles.

84. If the number of sides of a regular polygon inscribed in a circle be increased indefinitely, the apothem of the polygon will approach the radius of the circle as its limit.

85. The circumference of a circle is the limit which the perimeters of regular inscribed and circumscribed polygons approach when the number of their sides is increased indefinitely; and the area of the circle is the limit of the areas of these polygons.

86. The circumferences of two circles are to each other as their radii.

87. The circumference of a circle is equal to the product of its radius by twice the constant number π:

$$C = 2\pi r.$$

88. The area of a circle is equal to half the product of its circumference by its radius; or to the product of the square of its radius by the constant number π:

$$A = \pi r^2.$$

89. The areas of two circles are to each other as the squares of their radii.

CONSTRUCTIONS

1. To bisect a given line-segment.
2. To bisect a given angle.
3. To draw a perpendicular to a given line through a given external point.
4. At a given point of a given line, to erect a perpendicular to the line.
5. To construct an angle equal to a given angle.
6. Through a given point to draw a straight line parallel to a given straight line.
7. To divide a given line-segment into a given number of equal parts.
8. To construct a triangle, when the three sides are given.
9. To construct a triangle, when two sides and the included angle are given.
10. To construct a triangle, when two angles and a side are given.
11. To circumscribe a circle about a given triangle.
12. To inscribe a circle in a given triangle.
13. To draw a tangent to a circle through a given point on the circle.
14. To draw a tangent to a circle through a given external point.
15. To divide a line-segment into parts proportional to given line-segments.
16. To construct the fourth proportional to three given line-segments.
17. To construct the mean proportional between two given line-segments.
18. On a given line-segment as side, to construct a polygon similar to a given polygon.
19. To inscribe a square in a given circle.
20. To inscribe a regular hexagon in a given circle.

SOLID GEOMETRY

BOOK VI

LINES AND PLANES IN SPACE

1. If two planes meet, they intersect in a straight line.

2*. If a line is perpendicular to each of two intersecting lines at their point of intersection, it is perpendicular to the plane of the two lines.

3. All the perpendiculars that can be drawn to a given line at a given point lie in a plane which is perpendicular to the given line at the given point.

4. Through a given point in a given line one plane, and only one, can be passed perpendicular to the line.

5. Through a given external point one plane, and only one, can be passed perpendicular to a given line.

6*. Through a given point in a plane there can be drawn one line perpendicular to the plane, and only one.

7*. Through a given external point there can be drawn one line perpendicular to a given plane, and only one.

8. The perpendicular is the shortest line from a point to a plane.

9. Oblique lines drawn from a point to a plane, meeting the plane at equal distances from the foot of the perpendicular, are equal; and of two oblique lines meeting the plane at unequal distances from the foot of the perpendicular, the one corresponding to the greater distance is the greater.

10*. Two lines perpendicular to the same plane are parallel.

11. If one of two parallel lines is perpendicular to a plane, the other is also perpendicular to the plane.

12. If two lines are parallel to a third line, they are parallel to each other.

13. If two lines are parallel, every plane containing one of the lines, and only one, is parallel to the other.

14. Through either of two skew lines one plane, and only one, can be passed parallel to the other line.

15. Through a given point in space one plane, and only one, can be passed parallel to each of two skew lines, or else parallel to one line and containing the other.

16. Two planes perpendicular to the same line are parallel.

17. If two parallel planes are cut by a third plane, the lines of intersection are parallel.

18. A line perpendicular to one of two parallel planes is perpendicular to the other also.

19. Through a given point outside a plane one plane, and only one, can be passed parallel to the given plane.

20. If two intersecting lines are each parallel to a plane, the plane of these lines is parallel to that plane.

21*. If two angles not in the same plane have their sides respectively parallel and in the same sense, they are equal and their planes are parallel.

22. If two lines are cut by three parallel planes, their corresponding segments are proportional.

Dihedral Angles

23*. If two planes are perpendicular to each other, a line drawn in one of them perpendicular to their intersection is perpendicular to the other.

24. If two planes are perpendicular to each other, a line perpendicular to one of them at any point of their intersection will lie in the other.

25. If two planes are perpendicular to each other, a line drawn perpendicular to one of them through any point of the other will lie in the second plane.

26*. If a line is perpendicular to a given plane, every plane which contains this line is perpendicular to the given plane.

27*. If each of two intersecting planes is perpendicular to a third plane, their intersection is also perpendicular to that plane.

28. Through a given line not perpendicular to a given plane, one plane and only one can be passed perpendicular to the given plane.

29. Between any two given skew lines there is one common perpendicular, and only one.

Loci

30. The locus of points equidistant from two given points is the plane perpendicular to the line joining them, at its midpoint.

31. The locus of points equidistant from the vertices of a triangle is the line through the center of the circumscribed circle, perpendicular to the plane of the triangle.

32. The locus of points equidistant from two given intersecting planes is the pair of planes bisecting the dihedral angles formed by the given planes.

Polyhedral Angles

33. The sum of any two face angles of a trihedral angle is greater than the third face angle.

34. The sum of the face angles of any convex polyhedral angle is less than four right angles.

35. There cannot be more than five regular convex polyhedrons.

36. Two trihedral angles are equal or symmetric if the three face angles of one are equal respectively to the three face angles of the other.

BOOK VII

PRISMS AND PYRAMIDS. CYLINDERS AND CONES

Prisms and Parallelepipeds

37*. The sections of a prism made by parallel planes cutting all the lateral edges are congruent polygons.

38. Two prisms are congruent if three faces which include a trihedral angle of one are respectively congruent to three faces which include a trihedral angle of the other, and are similarly placed.

39. Two right prisms having congruent bases and equal altitudes are congruent.

40*. An oblique prism is equivalent to a right prism whose base is a right section of the oblique prism, and whose altitude is equal to a lateral edge of the oblique prism.

41*. The lateral area of a prism is equal to the product of a lateral edge by the perimeter of a right section.

42. The opposite faces of a parallelepiped are congruent.

43*. The plane passed through two diagonally opposite edges of a parallelepiped divides it into two equivalent triangular prisms.

44. The volume of any parallelepiped is equal to the product of its base by its altitude.

45*. The volume of a triangular prism is equal to the product of its base by its altitude.

46. The volume of any prism is equal to the product of its base by its altitude.

Pyramids

47*. If a pyramid is cut by a plane parallel to the base, the lateral edges and the altitude are divided proportionally, and the section is a polygon similar to the base.

48*. The lateral area of a regular pyramid is equal to half the product of its slant height by the perimeter of its base:

$$A = \tfrac{1}{2}lp.$$

49. The lateral area of a frustum of a regular pyramid is equal to half the sum of the perimeters of the bases multiplied by the slant height of the frustum:

$$A = \tfrac{1}{2}l(p+p').$$

50. Two triangular pyramids having equivalent bases and equal altitudes are equivalent.

51*. The volume of a triangular pyramid is equal to one-third the product of its base by its altitude.

52. The volume of any pyramid is equal to one-third the product of its base by its altitude.

Cylinders

53. The bases of a cylinder are congruent.

54. The lateral area of a circular cylinder is equal to the product of an element by the perimeter of a right section.

For a cylinder of revolution:

$$A = 2\pi rh.$$

55. The volume of a circular cylinder is equal to the product of its base by its altitude.

For a cylinder of revolution:

$$V = \pi r^2 h.$$

Cones

56*. In a circular cone, a section made by a plane parallel to the base is a circle.

57. The lateral area of a cone of revolution is equal to half the product of its slant height by the circumference of its base:

$$A = \pi rl.$$

58. The lateral area of a frustum of a cone of revolution is equal to half the sum of the circumferences of the bases multiplied by the slant height of the frustum:

$$A = \pi l(r + r').$$

59. The volume of a circular cone is equal to one-third the product of its base by its altitude:

$$V = \tfrac{1}{3}\pi r^2 h.$$

Similar Solids

60. The areas of two similar polyhedrons are to each other as the squares of any two corresponding edges.

61. The volumes of two similar polyhedrons are to each other as the cubes of any two corresponding edges.

62. The areas of two similar cylinders, or of two similar cones, are to each other as the squares of any two corresponding lines.

63. The volumes of two similar cylinders, or of two similar cones, are to each other as the cubes of any two corresponding lines.

BOOK VIII

THE SPHERE

64*. If a plane intersects a sphere, the intersection is a circle.

65. Through any two given points on the surface of a sphere an arc of a great circle can be drawn.

66. Through any three given points on the surface of a sphere one circle and only one can be drawn.

67. The spherical distances of all points on a circle of a sphere from a given pole of the circle are equal.

68*. On any sphere, a point which is at the distance of a quadrant from each of two other points, not the extremities of a diameter, is a pole of the great circle passing through these points.

69. A plane perpendicular to a radius at its outer extremity is tangent to the sphere.

70. The plane tangent to a sphere at a given point is perpendicular to the radius drawn to that point.

71. A sphere can be inscribed in any given tetrahedron.

72*. A sphere can be circumscribed about any given tetrahedron.

73*. An angle formed by arcs of two great circles is measured by the arc of a great circle described from its vertex as a pole and included between its sides, produced if necessary.

Spherical Triangles and Polygons

74. Any side of a spherical triangle is less than the sum of the other two sides.

75. The sum of the sides of a convex spherical polygon is less than 360°.

76*. If one spherical triangle is the polar triangle of another, then reciprocally the second is the polar triangle of the first.‡

77*. In two polar triangles, any angle of either is the supplement of the opposite side of the other.‡

78*. The sum of the angles of a spherical triangle is greater than 180° and less than 540°.

79. Two symmetric spherical triangles are equivalent.

80. Two spherical triangles on the same sphere or on equal spheres are either congruent or symmetric if two sides and the included angle of one are equal respectively to two sides and the included angle of the other.

81. Two spherical triangles on the same sphere or on equal spheres are either congruent or symmetric if a side and the adjacent angles of one are equal respectively to a side and the adjacent angles of the other.

82. Two spherical triangles on the same sphere or on equal spheres are either congruent or symmetric if the three sides of one are equal respectively to the three sides of the other.

83. Two spherical triangles on the same sphere or on equal spheres are either congruent or symmetric if the three angles of one are equal respectively to the three angles of the other.

84. If two sides of a spherical triangle are equal, the angles opposite these sides are equal.

85. If two angles of a spherical triangle are equal, the sides opposite these angles are equal.

86. If two angles of a spherical triangle are unequal, the sides opposite these angles are unequal, and the side opposite the greater angle is the greater.

87. If two sides of a spherical triangle are unequal, the angles opposite these sides are unequal, and the angle opposite the greater side is the greater.

88. A spherical triangle is equivalent to a lune whose angle is half the spherical excess of the triangle.

89. A convex spherical polygon is equivalent to a lune whose angle is half the spherical excess of the polygon.

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‡ In Theorems 76 and 77, it is understood that the customary treatment in the textbooks, based on a figure in which one triangle is drawn inside the other, is all that is required.

The Measurement of the Sphere.

90. The shortest line that can be drawn on the surface of a sphere, connecting two given points of the sphere, is an arc of a great circle.

91. The area of a sphere is equal to the area of four great circles:

$$A = 4\pi r^2.$$

92. The volume of a sphere is equal to one-third the product of its radius by the area of its surface:

$$V = \tfrac{4}{3}\pi r^3.$$

PLANE AND SOLID GEOMETRY

MINOR REQUIREMENT

BOOK I

TRIANGLES, PERPENDICULARS, AND PARALLELS

Triangles

1*. Two triangles are congruent if two sides and the included angle of one are equal respectively to two sides and the included angle of the other.

2*. Two triangles are congruent if a side and the adjacent angles of one are equal respectively to a side and the adjacent angles of the other.

3*. If two sides of a triangle are equal, the angles opposite these sides are equal.

4*. If two angles of a triangle are equal, the sides opposite these angles are equal.

5*. Two triangles are congruent if the three sides of one are equal respectively to the three sides of the other.

Parallels and Perpendiculars

6*. Not more than one perpendicular can be drawn to a given line from a given external point.

7. Two lines in the same plane perpendicular to the same line are parallel.

8. If a line is perpendicular to one of two parallel lines, it is perpendicular to the other also.

9. Two right triangles are congruent if the hypotenuse and an adjacent angle of one are equal respectively to the hypotenuse and an adjacent angle of the other.

10*. Two right triangles are congruent if the hypotenuse and a side of one are equal respectively to the hypotenuse and a side of the other.

11*. If two parallel lines are cut by a transversal, the alternate interior angles are equal.

12. If two parallel lines are cut by a transversal, the corresponding angles are equal.

13. If two lines are cut by a transversal so that a pair of alternate interior angles are equal, the lines are parallel.

14. If two lines are cut by a transversal so that a pair of corresponding angles are equal, the lines are parallel.

15. If two angles in a plane have their sides respectively parallel or respectively perpendicular, they are either equal or supplementary.

Parallelograms

16. The opposite sides of a parallelogram are equal, and the opposite angles are equal.

17. The diagonals of a parallelogram bisect each other.

18*. If the opposite sides of a plane quadrilateral are equal, the figure is a parallelogram.

19*. If two sides of a quadrilateral are equal and parallel, the figure is a parallelogram.

20*. If three or more parallels intercept equal segments on one transversal, they intercept equal segments on every transversal.

Sum of the Angles of a Triangle

21*. The sum of the three angles of a triangle is equal to two right angles.

22. An exterior angle of a triangle is equal to the sum of the two opposite interior angles.

Inequalities

23. If two sides of a triangle are unequal, the angles opposite these sides are unequal, and the angle opposite the greater side is the greater.

24. If two angles of a triangle are unequal, the sides opposite these angles are unequal, and the side opposite the greater angle is the greater.

Loci

25*. In a plane, the locus of points equidistant from two given points of the plane is the perpendicular bisector of the line joining them.

26*. In a plane, the locus of points equidistant from two given intersecting lines in the plane is the pair of lines bisecting the angles formed by the given lines.

BOOK II

THE CIRCLE

27*. Through any three given points not lying in a straight line one circle, and only one, can be drawn.

28. In the same circle, or in equal circles, equal central angles intercept equal arcs, and equal arcs determine equal central angles.

29. In the same circle, or in equal circles, two central angles are proportional to their intercepted arcs.

30. In the same circle, or in equal circles, if two arcs are equal, their chords are equal; and, conversely, equal chords determine equal minor arcs and equal major arcs.

31*. In the same circle, or in equal circles, equal chords are equidistant from the center, and chords equidistant from the center are equal.

32*. A diameter perpendicular to a chord of a circle bisects the chord and the arcs determined by the chord.

33. A diameter which bisects a chord (that is not a diameter) is perpendicular to it.

Tangents

34. A line perpendicular to a radius at its outer extremity is tangent to the circle.

35. The tangent to a circle at a given point is perpendicular to the radius drawn to that point.

36. If tangents to a circle from an external point are drawn, they make equal angles with the line joining the given point to

the center, and the segments from the given point to the points of contact are equal.

37. Two parallel secants, or two parallel tangents, or a tangent and a secant parallel to it, intercept equal arcs on a circle.

The Measure of Angles

38*. An inscribed angle is measured by half the intercepted arc.

39. Angles inscribed in the same segment, or in equal segments, are equal.

40. An angle inscribed in a semicircle is a right angle.

41. An angle formed by a tangent and a chord of a circle is measured by half the intercepted arc.

BOOK III

SIMILAR POLYGONS

Similar Triangles

42. A line parallel to one side of a triangle and intersecting the other two sides divides these two sides proportionally.

43*. If a line divides two sides of a triangle proportionally, it is parallel to the third side.

44*. Two triangles are similar if the angles of one are respectively equal to the angles of the other; or if two angles of one are respectively equal to two angles of the other.

45*. Two triangles are similar, if an angle of one equals an angle of the other and the sides including these angles are proportional.

46*. Two triangles are similar if their sides are respectively proportional.

Applications

47. The bisector of an (interior) angle of a triangle divides the opposite side into segments which are proportional to the adjacent sides.

48. The segments cut off on two transversals by a series of parallels are proportional.

49. In any right triangle, the perpendicular dropped from the vertex of the right angle to the hypotenuse divides the triangle into two triangles, each similar to the given triangle.

50*. In any right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Similar Polygons

51. The perimeters of two similar polygons have the same ratio as any two corresponding sides.

BOOK IV

AREAS OF POLYGONS

52*. The area of a parallelogram is equal to the product of its base by its altitude:

$$A = bh.$$

53. The area of a triangle is equal to half the product of its base by its altitude:

$$A = \tfrac{1}{2}bh.$$

54. Two parallelograms, or two triangles, having equal bases and equal altitudes are equivalent.

55. The area of a trapezoid is equal to half the product of the sum of its bases by its altitude:

$$A = \tfrac{1}{2}(b + b')h.$$

56. The areas of two similar triangles are to each other as the squares of any two corresponding sides.

57. The areas of two similar polygons are to each other as the squares of any two corresponding sides.

BOOK V

THE MEASURE OF THE CIRCLE

58. The circumference of a circle is equal to the product of its radius by twice the constant number π:

$$C = 2\pi r.$$

59. The area of a circle is equal to half the product of its circumference by its radius; or to the product of the square of its radius by the constant π:

$$A = \pi r^2.$$

BOOK VI

LINES AND PLANES IN SPACE

60. If two planes meet, they intersect in a straight line.

61*. If a line is perpendicular to each of two intersecting lines at their point of intersection, it is perpendicular to the plane of the two lines.

62. All the perpendiculars that can be drawn to a given line at a given point lie in a plane which is perpendicular to the given line at the given point.

63. Through a given point in a given line one plane, and only one, can be passed perpendicular to the line.

64. Through a given external point one plane, and only one, can be passed perpendicular to a given line.

65*. Through a given point in a plane there can be drawn one line perpendicular to the plane, and only one.

66*. Through a given external point there can be drawn one line perpendicular to a given plane, and only one.

67. The perpendicular is the shortest line from a point to a plane.

68*. Two lines perpendicular to the same plane are parallel.

69. If one of two parallel lines is perpendicular to a plane, the other is also perpendicular to the plane.

70. If two lines are parallel to a third line, they are parallel to each other.

71. If two lines are parallel, every plane containing one of the lines, and only one, is parallel to the other line.

72. Through either of two skew lines one plane, and only one, can be passed parallel to the other line.

73. Through a given point in space one plane, and only one, can be passed parallel to each of two skew lines, or else parallel to one line and containing the other.

74. Two planes perpendicular to the same line are parallel.

75. If two parallel planes are cut by a third plane, the lines of intersection are parallel.

76. A line perpendicular to one of two parallel planes is perpendicular to the other also.

77. Through a given point outside a plane one plane, and only one, can be passed parallel to the given plane.

78. If two intersecting lines are each parallel to a plane, the plane of these lines is parallel to that plane.

79*. If two angles not in the same plane have their sides respectively parallel and in the same sense, they are equal and their planes are parallel.

Dihedral Angles

80*. If two planes are perpendicular to each other, a line drawn in one of them perpendicular to their intersection is perpendicular to the other.

81. If two planes are perpendicular to each other, a line perpendicular to one of them at any point of their intersection will lie in the other.

82. If two planes are perpendicular to each other, a line drawn perpendicular to one of them through any point of the other will lie in the second plane.

83*. If a line is perpendicular to a given plane, every plane which contains this line is perpendicular to the given plane.

84*. If each of two intersecting planes is perpendicular to a third plane, their intersection is also perpendicular to that plane.

85. Through a given line not perpendicular to a given plane, one plane and only one can be passed perpendicular to the given plane.

86. Between any two given skew lines there is one common perpendicular, and only one.

BOOK VII

PRISMS AND PYRAMIDS. CYLINDERS AND CONES

Prisms and Parallelepipeds

87. The volume of any parallelepiped is equal to the product of its base by its altitude.

88. The volume of any prism is equal to the product of its base by its altitude.

Pyramids

89*. If a pyramid is cut by a plane parallel to the base, the lateral edges and the altitude are divided proportionally, and the section is a polygon similar to the base.

90. The volume of any pyramid is equal to one-third the product of its base by its altitude.

Cylinders

91. The lateral area of a circular cylinder is equal to the product of an element by the perimeter of a right section.

For a cylinder of revolution:

$$A = 2\pi rh.$$

92. The volume of a circular cylinder is equal to the product of its base by its altitude.

For a cylinder of revolution:

$$V = \pi r^2 h.$$

Cones

93. The lateral area of a cone of revolution is equal to half the product of its slant height by the circumference of its base:

$$A = \pi rl.$$

94. The volume of a circular cone is equal to one-third the product of its base by its altitude:

$$V = \tfrac{1}{3}\pi r^2 h$$

BOOK VIII

THE SPHERE

95. The area of a sphere is equal to the area of four great circles:

$$A = 4\pi r^2.$$

96. The volume of a sphere is equal to one-third the product of its radius by the area of its surface:

$$V = \tfrac{4}{3}\pi r^3.$$

CONSTRUCTIONS

1. To bisect a given line-segment.
2. To bisect a given angle.
3. To draw a perpendicular to a given line through a given external point.
4. At a given point of a given line, to erect a perpendicular to the line.
5. To construct an angle equal to a given angle.
6. Through a given point to draw a straight line parallel to a given straight line.
7. To divide a given line-segment into a given number of equal parts.
8. To construct a triangle, when the three sides are given.
9. To construct a triangle, when two sides and the included angle are given.
10. To construct a triangle, when two angles and a side are given.
11. To circumscribe a circle about a given triangle.
12. To inscribe a circle in a given triangle.
13. To draw a tangent to a circle through a given point on the circle.

14. To draw a tangent to a circle through a given external point.
15. To divide a line-segment into parts proportional to given line-segments.
16. To construct the fourth proportional to three given line-segments.
17. To construct the mean proportional between two given line-segments.
18. To inscribe a square in a given circle.
19. To inscribe a regular hexagon in a given circle.

APPENDIX

FORMULAS OF MENSURATION FOR SOLIDS

$S =$ area of curved surface; $B =$ area of base of a solid; $h =$ altitude; $V =$ volume; $l =$ slant height; $r =$ radius of a circle or sphere.

1. Formulas for the areas of the surfaces of polyhedrons and for the total surface of a cone and cylinder should not be memorized. Every problem of this type should be solved by finding the sum of the areas of the constituent faces.

2. The following formulas should be memorized:

Prism and cylinder:	$V = Bh.$	
Pyramid and cone:	$V = \frac{1}{3}Bh.$	
Right circular cylinder:	$S = 2\pi rh$;	$V = \pi r^2 h.$
Right circular cone:	$S = \pi rl$;	$V = \frac{1}{3}\pi r^2 h.$
Sphere:	$S = 4\pi r^2$;	$V = \frac{4}{3}\pi r^3.$
Zone:	$S = 2\pi rh.$	

3. The candidate should be familiar with the following formulas to the extent of being able to use them when they are given to him. It is not expected that he will memorize them.

Frustum of a pyramid or cone:

$$V = \tfrac{1}{3}h(B + \sqrt{BB'} + B').$$

Frustum of a right circular cone:

$$S = \pi l(r + r'); \qquad V = \tfrac{1}{3}\pi h(r^2 + rr' + r'^2).$$

Spherical sector: $V = \frac{1}{3}rS$ ($S =$ area of zone).

A spherical segment of one base should be treated as the difference between the spherical sector and the cone.

DRAWING

An important aid to the visualization of space figures is the ability to draw these figures on paper. The pupil should be trained to make neat free-hand drawings, inserting whatever construction lines are needed and supplementing the representation of the space figure by independent drawings of plane sections, whenever such contribute to clearness. Ruler and compasses may often be used with advantage, but no technical knowledge of descriptive geometry or the niceties of mechanical drawing forms any part of the requirement. The use of ruler and compasses in the examination is permitted and, to a moderate degree, desirable; but it is not prescribed.

The accompanying drawings indicate the sort of space figures which, beside those commonly met in the text-books, the candidate may need to represent on paper in connection with the problems which appear below.

Figure 1. Figure (*a*) represents a right parallelepiped, the base of which is a rhombus. Figure (*b*) represents one quarter of the parallelepiped, differently placed, the three angles at O representing right angles. Figure (*c*) shows the base with the correct proportions of the actual solid.

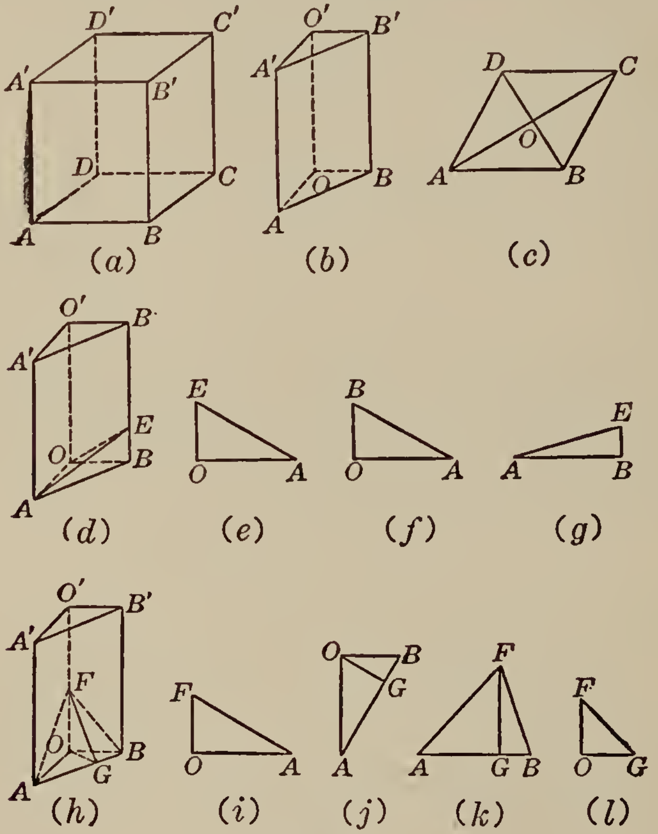
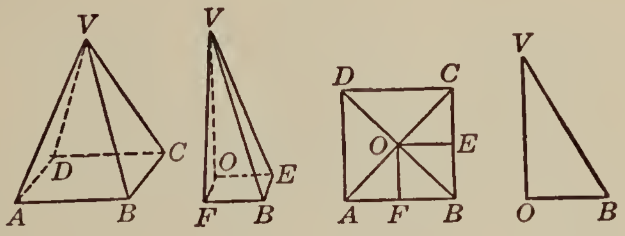


FIG. 1.

Figure (*d*) is a representation of a section of the prism (*b*) by a plane passing through OA and cutting BB' at E. Figures (*e*), (*f*), (*g*) show triangles of Figure (*d*) drawn with the correct proportions of the actual solid. It is unnecessary to draw a separate figure of triangle OEB since it appears with correct proportions in Figure (*d*).

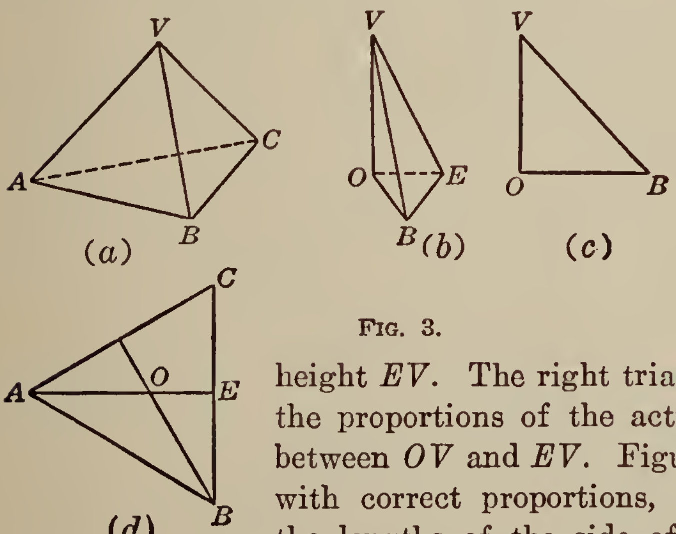
Figure (*h*) is a representation of a section of the prism (*b*) by a plane passing through *AB* and cutting *OO′* at *F*. *OGF* is a plane angle of the dihedral angle *O-AB-F*. Figures (*i*), (*j*), (*k*), (*l*) show, with correct proportions, those triangles of Figure (*h*) which are distorted by projection.

Figure 2. Figure (*a*) represents a regular pyramid with square base. Figure (*b*) represents one quarter of the pyramid and shows the altitude *OV* and the slant height *EV* or *FV*. The relations between the lengths of the altitude, the slant height, and the edge of the base are shown with correct proportions in the triangle *OEV*. Figures (*c*) and (*d*) show the plane figures necessary for an understanding of the relations between the lengths of the side of the base, the lateral edge, and the altitude.



(*a*) (*b*) (*c*) (*d*)

FIG. 2.

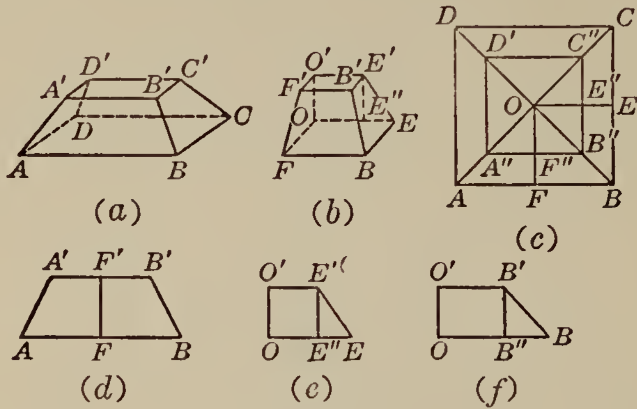


(*a*) (*b*) (*c*) (*d*)

FIG. 3.

Figure 3. Figure (*a*) represents a regular pyramid with a triangular base. Figure (*b*) represents one sixth of the pyramid and shows the altitude *OV* and the slant height *EV*. The right triangle *OEV* shows, with the proportions of the actual solid, the relation between *OV* and *EV*. Figures (*c*) and (*d*) show, with correct proportions, the relations between the lengths of the side of the base, the lateral edge, and the altitude.

Figure 4. Figure (*a*) represents a frustum of a regular pyramid with a square base. Figure (*b*) represents one quarter of the frustum, showing the altitude OO' and the slant height EE' or FF'.

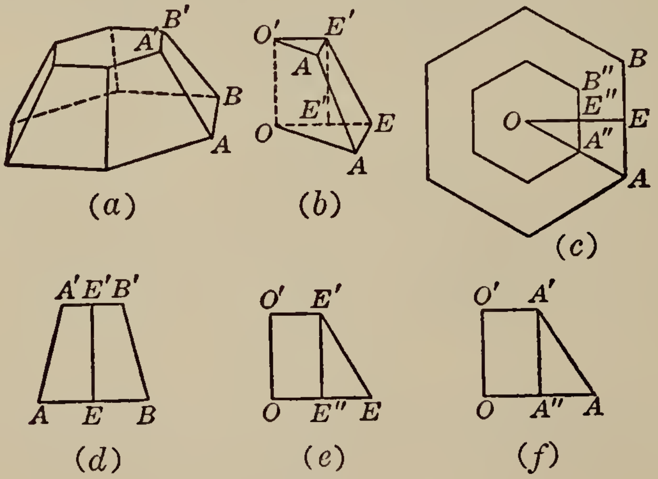


(*a*) (*b*) (*c*) (*d*) (*e*) (*f*)

FIG. 4.

Figure (*c*) represents the lower base on which the upper base has been projected orthogonally. Figures (*d*), (*e*), and (*f*) represent plane figures selected to show, with correct proportions, the relations between the lengths of the sides of the upper and lower bases, the slant height, the altitude, and the lateral edge.

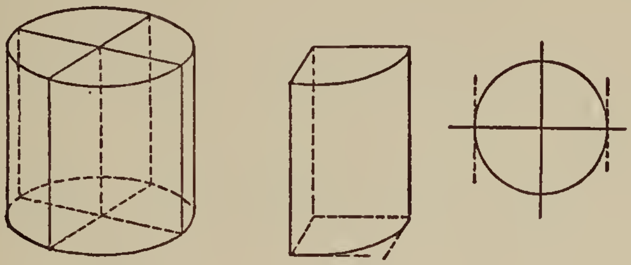
Figure 5. Figure (*a*) represents a frustum of a regular pyramid with a hexagonal base. Figure (*b*) represents one twelfth of the frustum, showing the altitude OO' and the slant height EE'. Figure (*c*) represents the lower base, on which the upper base has been projected orthogonally. Figures (*d*), (*e*), and (*f*) represent plane figures selected to show, with the proportions of the actual solid, the relations between the lengths of the sides of the upper and lower bases, the slant height, the altitude, and the lateral edge.



(*a*) (*b*) (*c*) (*d*) (*e*) (*f*)

FIG. 5.

Figure 6. Figure (*a*) represents a right circular cylinder divided into four equal parts by two planes at right angles. It is frequently possible to use but one-fourth of the cylinder, which may be drawn as in Figure (*b*).



(*a*) (*b*) (*c*)

Fig. 6.

When, however, Figure 6(*a*) is needed, it is well to draw it free-hand, including only such construction lines as are required. The arcs of ellipses of Figure 6(*b*) may be drawn free-hand. If care be taken that they have the right directions at their extremities as indicated by the dotted lines, it is not difficult to draw them with sufficient accuracy to make the figure look right.

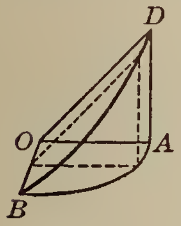
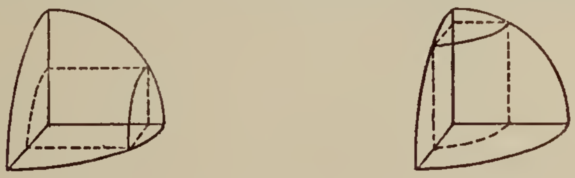
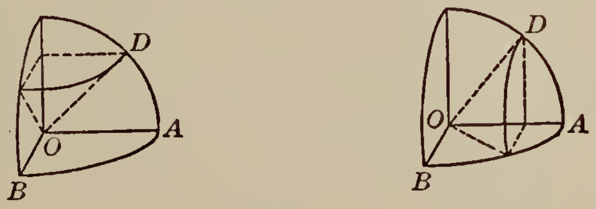


Fig. 7.

Figures 7, 8, *and* 9. In representing the intersection of surfaces it is usually sufficient to show only a portion of the curve of intersection. Figure 7 represents the intersection of a cylindrical surface and a plane meeting the axis at an angle of 45°. Figures 8(*a*) and 8(*b*) represent the intersection of a cylindrical surface and a spherical surface, the center of the sphere lying on the axis of the cylinder. Figures 9(*a*) and 9(*b*) represent the intersection of a conical surface of vertical angle 90° with a spherical surface, the vertex of the cone being at the center of the sphere.



(*a*) Fig. 8. (*b*)



(*a*) Fig. 9. (*b*)

PROBLEMS

The following problems are intended to show the kind of training expected of a candidate in solid geometry. They should not be looked upon as typical examination questions.

The problems in mensuration involve a thorough understanding of the relations of lines and planes as treated in Book VI, and involve solids for which the calculations depend upon the ability to recognize and deal with right and isosceles triangles in the space figures. See Examples 1, 2, 3, 4, 6, 7, 8, 9.

Mensuration of pyramids is not limited to regular pyramids and the frustums of regular pyramids. See Examples 4, 6, 7, 8.

The candidate should be able to calculate by means of the tables the area of a regular pentagon, hexagon, and octagon. See Examples 2, 9, 10, 19.

The relations between linear dimensions, areas, and volumes of similar solids should be thoroughly familiar to the candidate. See Examples 5, 13, 14, 15, 16.

In connection with the sphere the candidate will be expected to solve problems in mensuration involving inscribed and circumscribed cylinders, cones, regular prisms, and regular pyramids. See Examples 17, 18, 19.

The candidate should be able to describe accurately and completely loci involving intersections of the standard loci mentioned in Propositions 30, 31, 32, to which should be added the loci of points at a given distance (*a*) from a fixed point; (*b*) from a fixed line; (*c*) from a fixed plane.

1. The base of a right parallelepiped is a rhombus of side 12.3″ and acute angle 55°. The length of the lateral edge is 16.8″. Compute the area of the total surface; or compute the length of the longest diagonal; or compute the volume.

2. Calculate the volume (or the area of the total surface) of a regular pyramid with triangular (or square, or pentagonal, or hexagonal, or octagonal) base if the side of the base is 10″ and the lateral edge is 20″.

3. The figure represents a truncated prism with a square base. The lateral edges are perpendicular to the base. $AB = 10''$, $AE = 6''$, $BF = 10''$, $CG = 10''$, $DH = 6''$. Compute the length of the diagonal FD; or compute the number of degrees in the face angle EFB; or compute the area of each face; or compute the number of degrees in the angle BFH; or compute the volume.

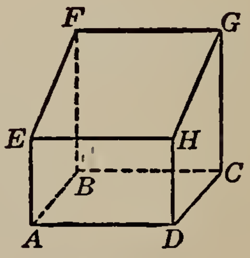


FIG. 10.

4. The base of a pyramid is a rectangle $ABCD$. The lateral edge VA is perpendicular to the base. $AB=6''$, $BC=8''$, $VA=10''$. Compute the number of degrees in any given face angle; or compute the number of degrees in the angle which BV (or CV or DV) makes with the plane $ABCD$; or compute the number of degrees in the dihedral angles formed by $ABCD$ with each of the other four planes; or calculate the area of the total surface.

5. If the pyramid of Example 4 is made of solid wood and weighs 5 pounds how far from the vertex must it be cut by a plane parallel to the base so that the frustum cut off will weigh 3 pounds?

6. A frustum of the pyramid of Example 4 is formed by a plane $6''$ from the base. Compute the number of degrees in any given face angle; or compute the area of the total surface; or compute the volume, given

$$V=\tfrac{1}{3}h(B+\sqrt{BB'}+B').$$

7. In the figure, $ABCDA'B'C'D'$ is a cube of edge $12''$ and V is the mid-point of $A'B'$. Compute the lengths of VA, VB, VC, and VD; or compute the number of degrees in the angles of the triangle VAB (or VBC or VCD). What angle in the figure is a plane angle of the dihedral angle formed by $ABCD$ with VBC; with VAD? Explain a method for constructing a plane angle of the dihedral angle formed by $ABCD$ with VCD. How many degrees are there in this angle? Compute the area of each face of the pyramid.

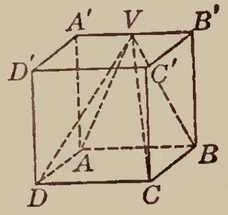


FIG. 11.

8. A frustum of the pyramid of Example 7 is formed by a plane $6''$ from the base. Compute the number of degrees in any given face angle of the frustum; or compute the area of the total surface of the frustum; or compute the volume of the frustum, given

$$V=\tfrac{1}{3}h(B+\sqrt{BB'}+B').$$

9. If the side of the lower base is $7'$, the side of the upper base is $3'$, and the altitude is $4'$, compute the volume (or the area of the total surface) of the frustum of a regular pyramid with triangular (or square, or pentagonal, or hexagonal, or octagonal) base.

$$V=\tfrac{1}{3}h(B+\sqrt{BB'}+B').$$

10. If the side of the lower base is 10″, the side of the upper base is 5″, and the lateral edge is 10″, compute the volume (or the area of the total surface) of the frustum of a regular pyramid with triangular (or square, or pentagonal, or hexagonal, or octagonal) base.

$$V = \tfrac{1}{3}h(B + \sqrt{BB'} + B').$$

11. A vat, open at the top, is to be built in the form of a frustum of a cone to contain 1000 cu. ft. The diameter of the upper base is to be 12′ and of the lower base 8′. Calculate the number of square feet of galvanized iron required, assuming that 15% of the iron purchased will be wasted or used for joints.

$$S = \pi l(r + r'); \qquad V = \tfrac{1}{3}\pi h(r^2 + rr' + r'^2).$$

12. A round tower has a roof in the form of a frustum of a cone, the sloping sides of which make an angle of 45° with the horizontal. The diameter at the bottom is 31.5 ft. and at the top is 15.5 ft. Calculate the number of square feet in the roof of the tower.

$$S = \pi l(r + r').$$

13. Two lead balls, each 1 in. in diameter, are melted up and cast into a single sphere. What is the diameter of the latter?

14. If a spherical drop of water is vaporized into a spray consisting of a thousand equal spheres without altering the volume, by what percent has the total surface been increased?

15. The area of the entire surface of a solid polyhedron weighing 64 lbs. is 340 sq. in. What is the surface of a similar polyhedron made of the same material and weighing 1000 lbs.?

16. It is desired to double the capacity of a spherical balloon. By what percentage must the area of canvas be increased?

17. Find the volume of the largest sphere which can be cut from a metal cone, the diameter of whose base is 6″ and whose slant height is 6″?

18. Find the surface and volume of a sphere circumscribing a cylinder of revolution, the radius of whose base is 6″ and whose altitude is 8″.

19. The diameter of a sphere is 18″. Calculate the volume of the largest regular prism (or pyramid) of altitude 15″ which can be cut from the sphere if the base of the prism (or pyramid) is a triangle (or square, or pentagon, or hexagon, or octagon).

20. The locus of points whose distances from a fixed plane and a fixed line perpendicular to the plane are in a given ratio is the surface of a cone of revolution having the given line for its axis and the point of intersection of the line and plane for its vertex.

21. P is a horizontal plane and Q is a vertical plane. A is a point in the line of intersection of P and Q. l is a line in Q perpendicular to P at A. m is a line in P perpendicular to Q at A. Describe the locus of points in space which are 5″ from A and 3″ from P; or 5″ from A and 3″ from l; or equidistant from P and Q; or equidistant from l and m; or 5″ from A and equidistant from P and Q; or 5″ from A and 3″ from Q.

22. In a fixed plane M a line l is drawn and a point A marked on l. Describe the locus of the points P in space (1) such that the perpendicular from P on l always makes an angle of 25° (or 7°, 48°, etc.) with M; or (2) such that AP always makes an angle of 18° (or 69°, 143°, etc.) with l.

(May, 1923—5 M)
(October, 1923—2 M)
(March, 1924—2 M)
(September, 1925—2 M)
(May, 1926—2½ M.)
(April, 1927—4½ M.)
(January, 1929—5 M)

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